Mathematics Done in English



Douglas Perkins June 29, 2016

Preface

"Number rules the universe."

- The Pythagoreans

"Mathematics is the only good metaphysics."

– Lord Kelvin

"Mathematics is the queen of the sciences, and arithmetic the queen of mathematics."

– C. F. Gauss

"A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery."

- Georg Polya, "How To Solve It" (1945)



Ravnen (1890)

I first wrote this textbook in 2015 for a tenth grade English class in Japan. My students are planning to study abroad in eleventh grade. A good way to learn to do math in English is to do math in English. Topics should be accessible but engaging. So, we make tangents into gambling, game theory, computers, art, and elsewhere if we like.

Many people helped me brainstorm and proofread. Marjorie Carlson, Adam Pearson, Meghan Sahara, Andrew Leung, and Betsy Perkins helped out a lot. I majored in math in college, and much of the love I have for the subject comes from those wonderful professors and classes. On the pump.io network, Efraim Flashner, JanKusanagi, and Stephen Sekula offered support and suggestions.

– Douglas Paul Perkins. Nishitokyo, Japan.

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1968 calculator. Photo by Michael Holley.





Part I: Numbers



Dice. Picture by CGP Grey.

Chapter 1: Counting

Telephone Interview

- Ask your classmates their telephone numbers.
- How fast can you say your own phone number?

The Unlucky 21 Game

- Make pairs.
- Start at 1 and count to 21.
- Each player can say 1, 2, 3, or 4 numbers.
- The person who says "21" loses.
- **Question**: The second player can always win. How?

Two Kinds of Numbers

Cardinals	one	two	three	four	five	six	
Ordinals	first	second	third	fourth	fifth	sixth	

Fill in the Blanks

- 1. Mark has _____ brother and no sisters.
- 2. The science room is on the _____ floor.
- 3. Five plus ten is _____.
- 4. Her birthday is on Halloween. That's October _____.
- 5. There are two outs and runners on _____, ____, and _____.

Comprehension

Practice listening to and saying the numbers.

A. 0	D. 1,024	G. 65,536	
B. 575	E. 1,112	H. 1,048,576	$(\cdot)^{(\cdot)}$
C37	F. 6,536	I. 10,048,576	÷ J



German telephone, 1972.

Card Counting

Take a deck of cards and total the numbers.

- Make a group. Go around clockwise.
- Start with zero.
- Flip over the top card.
- Add it to the total, and say the number.
- Repeat until all the cards are gone.
- Question: What is the total for the whole deck?

Sequences

What numbers come next?

A.	1	2	3	4	5	6	\rightarrow	 	
В.	2	4	6	8	10	12	\rightarrow	 	
C.	5	10	15	20	25	30	\rightarrow	 	
D.	1	3	9	27	81	243	\rightarrow	 	
Е.	2	3	5	7	11	13	\rightarrow	 	
F.	10	20	60	70	110	120	\rightarrow	 	
G.	3	1	4	1	5	9	\rightarrow	 	
H.	31	28	31	30	31	30	\rightarrow	 	

Sequence Making

Make some new sequences and show the class.

I.	 	 	 	\rightarrow	 	
J.	 	 	 	\rightarrow	 	

Minimal Pair Listening

Circle the word you hear.

1.	first	/	fast	6.	three /	tree	
2.	thirteen	/	thirty	7.	some /	sun	
3.	fourteen	/	forty	8.	third /	sad	
4.	nineteen	/	ninety	9.	big /	bag	
5.	tens	/	tense	10.	many /	money	



Matching

Pair the digits with the words.

1.	 eleven	A.	11
2.	 minus two	В.	110
3.	 negative five	C.	¹ / ₃
4.	 one hundred ten	D.	600,000
5.	 one half	E.	-2
6.	 one third	F.	0.51
7.	 six thousand	G.	6,000
8.	 zero point five one	H.	60,000
9.	 sixty thousand	I.	0.7
10.	 point seven	J.	²² / ₇
11.	 six hundred thousand	K.	-5
12.	 twenty-two over seven	L.	¹ / ₂

A Coin Game

- This is a two-player game: Player A and Player B.
- Player A goes first, then Player B, then Player A, and so on.
- Each turn, the player takes one coin from the table.
- Players can only choose coins on the left and right sides.
- The player with the most money at the end is the winner.
- Which player do you want to be?



Number Questions

1.	How much is five times itself?	25	55	85
2.	What number times itself equals four hundred?	10	18	20
3.	What is the largest even number less than ninety-nine?	10	98	100
4.	What number plus twice itself equals thirty-six?	8	12	16
5.	What number times anything equals itself?	0	1	π
6.	How much is one half plus one and a half?	2	2.5	3
7.	How many seconds are in two minutes?	120	200	220
8.	How many seconds are in an hour?	2,400	3,600	36,000

Roman Numerals

1	5	10	50	100	500	1000			
Ι	V	X	L	С	D	Μ			
	_			_	-	_	0	•	10

1	2	3	4	5	6	7	8	9	10
Ι	II	III	IV	\mathbf{V}	VI	VII	VIII	IX	X

Roman Numeral Practice

1.	How do you write <i>eleven</i> ?	
2.	How do you write <i>twelve</i> ?	
3.	How do you write <i>nine</i> ?	
4.	How do you write <i>nineteen</i> ?	
5.	How do you write <i>fourteen</i> ?	
6.	How do you write <i>fifty-five</i> ?	
7.	How do you write <i>159</i> ?	
8.	How do you write <i>1912</i> ?	
9.	How do you write <i>1982</i> ?	
10	How do you write <i>2016</i> ?	



Chapter 2: Chance







Flip a Coin

On average, heads and tails have the same chance of happening. Let's see what happens for us.

Flip #	1	2	3	4	5	6	7	8	9	10
Result	HEADS									
	TAILS									

Kinds of Dice

We commonly see dice with six sides. There are other kinds, though. Four, six, eight, ten, twelve, and twenty are common. Some board games and card games involve these unusual kinds of dice.



Probability

What is the probability of the following?

- A. Roll a d6 and get a 1.
- B. Roll a d8 and a d12 and get two 1s.
- C. Roll a d4 and a d20 and get two 4s.
- D. Roll three d6 dice and get all 6s.

 $P = \frac{1}{6}$ $P = \left(\frac{1}{8}\right) * \left(\frac{1}{12}\right) = \left(\frac{1}{96}\right)$ P =

 $P = _$



Roulette

Roulette is a gambling game. There is a wheel and a ball. Players make a bet on a number or pattern. The ball is rolled, and it stops on a number. If a player's bet is good, the person wins some money. If not, they lose some.

Math Questions

Write your answers.

- 1. If you bet \$1 on ① and you win, you get \$36.
 - What is the probability of a 1? _____
- 2. If you bet \$1 on (8) and you win, you get \$36.
 - What is the probability of an ⑧?
- 3. If you bet \$1 on RED, and you win, you get \$1.

What is the probability of an **RED**?

4. If you bet \$1 on EVEN and you win, you get \$1. (EVEN = 2, 4, 6, 8, ...)What is the probability of an EVEN? _____

Strategy Questions

Think about the answers.

- 1. Do you want to play Roulette?
- 2. If you play Roulette, do you want to win slowly or quickly? Why?

Chapter 3: Arithmetic

Drills

Answer the questions.

- 1. 5+5 = _____
- 2. $2 \times 16 =$ _____ 3. $64 \div 8 =$ _____
- 4. 12+24 = _____
- 5. 99-7 = _____
- 6. 2×2×2 = ____
- 7. $\sqrt{9} =$ ____
- 8. 10^{10} = _____

9.	Seven plus ten equals	·
10.	Nine times six equals	
11.	Four minus one equals	
12.	Thirty divided by five equals	
13.	One hundred minus one is	
14.	Twenty over four is	
15.	Five squared equals	
16.	The square root of sixteen is	



A graphing calculator.

Word Problems

Example	Mary bought ten red apples, three fruit does she have?	e green apples, and six or	anges. How m	any pieces of
	Expression:	10+3+6	Answer:	19 pieces
1.	Tom has three black T-shirts. He many shirts does he own?	has four more white T-shi	rts than black	T-shirts. How
	Expression:		Answer:	
2.	Aaron made thirty-six cookies. H does he have now?	le ate one and gave ten to	Beth. How mo	any cookies
	Expression:		Answer:	
3.	Brett studied for 30 minutes. The twenty minutes eating dinner. Ho	n he watched TV for an ho w long did that all take?	our. After that	, he spent
	Expression:		Answer:	

Word Problem Making

Make questions for the expressions.

Example	Expression:	50-(2×10)-3
	Question:	Max had \$50. He bought two \$10 watermelons. Then he bought a bag of oranges for \$3. How much money does he have now?
1.	Expression:	12-3-2
	Question:	
2.	Expression: Question:	0:30+1:00+0:15

6

Buy a Cupcake

A package of cupcakes costs \$3.58. How can you pay for it?



Answer	dollars	quarters	dimes	nickels	pennies	Total
Example	2	6	0	1	8	\$3.58
1.						
2.						
3.						
4.						

Chapter 4: Equations

Card Counting Revisited

In Chapter One, we summed up a deck of cards.

- **Question**: What was the total?
- **Question**: How can you calculate it?

Word Problems

Write the equation and solve it.

Example Joe's water bill costs \$5.00 a month for service, plus \$0.05 for each liter of water. Write an equation for **G**, his total monthly bill. Suppose he uses 73 liters of water in January. What is his January water bill?

Equation: $G = $5.00 + ($0.05 \times w)$ Solution: _____

1. The cost to rent a car is \$50.00 per day. Write an equation for **R**, rental costs. Meghan rented a car for five days. How much did she pay?

Equation: _____ Solution: _____

2. Danielle has a cell phone. The monthly service costs \$12.60. She pays an extra \$0.25 per minute. Write an equation for *C*, her total monthly bill. In December, her total bill was \$21.10. How many minutes did she talk?

Equation: _____ Solution: _____

3. Travis likes to buy apples and peaches. Apples cost \$0.75 each, and peaches cost \$1.12 each. Write an equation for **F**, his fruit bill. If he buys four apples and twice as many peaches, how much does it cost?

Equation: _____ Solution: _____

Comprehension

Practice listening to and saying the equations.

A. $y=x$	D. $y = \frac{1}{x}$	G. $a = (\frac{1}{2})bh$
B. $y=5x+2$	E. $y = \frac{2}{3x}$	H. $a=\pi r^2$
C. $y=ax+b$	F. $y=3x$	I. $A=4\pi r^2$



Number Brain Teasers

Write the missing words.

1.	12 = M in a Y.	M:	 Y:	
2.	26 = L of the A.	L:	 A:	
3.	52 = C in a D.	C:	 D:	
4.	7 = D in a W.	D:	 W:	
5.	4 = S in a Y.	S:	 Y:	
6.	47 = P in J.	P:	 J:	
7.	525,600 = M in a Y.	M:	 Y:	
8.	257 = C in the W.	C:	 W:	
9.	50 = S in the U.S.	S:	 U.S.:	
		2		

Make Your Own Brain Teaser



Break the Code

 23 5
 1 18 5
 23 8 1 20
 23 5
 20 8 9 14 11 .

 1 12 12
 20 8 1 20
 23 5
 1 18 5
 1 18 9 19 5 19

 23 9 20 8
 15 21 18
 20 8 15 21 7 8 20 19 .

 23 9 20 8
 15 21 18
 20 8 15 21 7 8 20 19 .

 23 9 20 8
 15 21 18
 20 8 15 21 7 8 20 19 .

 23 9 20 8
 15 21 18
 20 8 15 21 7 8 20 19 .

 23 5
 13 1 11 5
 20 8 5
 23 15 18 12 4 .

- Siddharta Gautama, 563-483 B.C.

Demonstrations I: Numbers

RULES

In pairs or individually, make and give a math demonstration. Presentations should be two to three minutes long. Explain the topic and show examples on the chalk board.

KEY POINTS

Clarity • Volume • Pronunciation • Chalkboard • Time

TOPICS

- A) Describe a foreign country's money system.
- B) What three neighboring numbers add up to 72?
- C) Explain how to read and write Mayan numerals.
- D) Explain how to read and write Babylonian numerals.
- E) Talk about Greek letters that are used in mathematics.
- F) Teach the class how to count to ten in a language they don't know.
- G) Two trains leave Budapest. One goes west at 50 km/h. The other goes east at 30 km/h. How far apart are they after two hours and ten minutes?
- H) Robert watched TV on Saturday and Sunday for a total of 4 hours. On Sunday he watched for twice as long as Saturday. How long did he watch on Saturday?
- I) Ken has an empty pool. It can hold 9000 buckets of water. He can carry 3 buckets a minute. Barbie can carry 2 buckets a minute. How long will it take to fill the pool?
- J) Jeff is 12 kg heavier than Scott. Their total mass is 143 kg. How much does Jeff weigh?



An Arabic clock. Photo by Looi.

Part II: Statistics





Chapter 5: School Statistics

Comprehension Listen and identify the charts. A. C. B. D. Three Kinds of Averages mean = $\frac{1+3+5}{3} = 3$ [1, 3, 5]**Mean**. The total divided by the count. **Median**. The middle number. [145,148,149,150,200] median = 149mode = 5[3, 5, 5, 7, 9, 15]**Mode**. The most common number. Calculate the Averages [4,8,9,12,20,20,21,27,28] [28, 28, 28, 32, 55, 56, 57, 59, 62] 1. 2. Mean: Mean: Median: _____ Median: _____ Mode: Mode: Minimal Pair Listening Circle the word you hear. 1. pie / buy 6. value / very 2. vase 7. base / thing / sing 3. chart / shirt 8. image / imagine product / produce 4. slice / slices 9. 5. cone / comb median / medium 10.

Interview Your Classmates

What size are your classmates' shoe sizes? Ask them and take notes here.



Calculate the Class Average Shoe Size

Mean:

Median: _____

Mode:



Average Height in History

Many people believe that the average height of humans is going up over the years, but actually it has varied over the centuries. Average height is affected by climate, cities, war, and population. One thousand years ago, the average height for men was 172.7 cm. In the 1600s and 1700s, it went down to 167 cm. In 2016 in America, the average for men was 178.2 cm and women was 164.1 cm. In the same year in Japan, the average for men was 171.2 cm and women was 158.8 cm.

1.	On average, men are taller than women.	TRUE / FALSE
2.	People were taller 1000 years ago than 400 years ago.	TRUE / FALSE
3.	American women are taller than American men.	TRUE / FALSE
4.	Human height has changed a lot in history.	TRUE / FALSE
5.	Human height has always gone up.	TRUE / FALSE

Chapter 6: Birthday Frequency

		Ар	ril 20	000					Ma	ay 20	000					Jui	ne 20	000					Ju	ly 20	00		
Su	М	Tu	W	Th	F	Sa	Su	М	Tu	W	Th	F	Sa	Su	М	Tu	M	Th	F	Sa	Su	М	Tu	W	Th	F	Sa
						1		1	2	3	4	5	6					1	2	3							1
2	3	4	5	6	7	8	7	8	9	10	11	12	13	4	5	6	7	8	9	10	2	3	4	5	6	7	8
9	10	11	12	13	14	15	14	15	16	17	18	19	20	11	12	13	14	15	16	17	9	10	11	12	13	14	15
16	17	18	19	20	21	22	21	22	23	24	25	26	27	18	19	20	21	22	23	24	16	17	18	19	20	21	22
23	24	25	26	27	28	29	28	29	30	31				25	26	27	28	29	30		23	24	25	26	27	28	29
30																					30	31					
		Aug	ust 2	2000		-		5	Septe	mber	· 200	0				Octo	ber	2000				ז	Nove	mbei	· 200)	
Su	М	Tu	W	Th	F	Sa	Su	М	Tu	W	Th	F	Sa	Su	М	Tu	W	Th	F	Sa	Su	М	Tu	W	Th	F	Sa
		1	2	3	4	5						1	2	1	2	3	4	5	6	7				1	2	3	4
6	7	8	9	10	11	12	3	4	5	6	7	8	9	8	9	10	11	12	13	14	5	6	7	8	9	10	11
13	14	15	16	17	18	19	10	11	12	13	14	15	16	15	16	17	18	19	20	21	12	13	14	15	16	17	18
20	21	22	23	24	25	26	17	18	19	20	21	22	23	22	23	24	25	26	27	28	19	20	21	22	23	24	25
27	28	29	30	31	20	20	24	25	26	27	28	29	30	29	30	31	20	20	27	20	26	27	28	29	30	24	20
			mbor	200	n				Ianu		2001					Fobr	1057	2001					Ma	rch 7	001		
0		Dece	mber	200	5	a .	0		Janu		2001	-	a .	0		reon	uary	2001		a .	0		IVIA		.001		a .
su	М	Tu	W	Th	E.	sa	su	M	Tu	W	Tn	E.	sa	Su	M	Tu	W	Th	E.	sa	su	М	Tu	W	Tn	E.	sa
		_		_	1	2	_	1	2	3	4	5	6		_		_	1	2	3		_		_	1	2	3
3	4	5	6	7	8	9	7	8	9	10	11	12	13	4	5	6	7	8	9	10	4	5	6	7	8	9	10
10	11	12	13	14	15	16	14	15	16	17	18	19	20	11	12	13	14	15	16	17	11	12	13	14	15	16	17
17	18	19	20	21	22	23	21	22	23	24	25	26	27	18	19	20	21	22	23	24	18	19	20	21	22	23	24
24	25	26	27	28	29	30	28	29	30	31				25	26	27	28				25	26	27	28	29	30	31
31																											

Birthdays of the Week

See the above chart to find the day of the week for your birthday. For example, if you were born on June first, 2000, you were born on a Thursday. Then, ask your classmates about their birthdays.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday		

Bar Graph

Make a bar graph showing the above information.



Birthday Months

Ask your classmates when their birthday is, and write it down.

January:	July:
February:	August:
March:	September:
April:	October:
May:	November:
June:	December:

Bar Graph

Make a bar graph showing the above information.



Average Birthday

Imagine Tom was born on May 12th and Jerry was born on August 8th. Forget about the month, and remember the number. The average of 12 and 8 is 10. Let's consider more people.

- What is the average day of this month?
- What is the average day of the month, considering all twelve months?
- Do you think the average birthday is on the average day of the month? Why?



Chapter 7: World Statistics



True/False Quiz

1.	There are more people from China than any other country.	TRUE / FALSE
2.	Brazilians are fewer in number than Americans.	True / False
3.	There are around 200,000 Filipinos.	TRUE / FALSE
4.	There are less than 50,000 Nepalese.	True / False
5.	There are about as many Americans as Peruvians.	TRUE / FALSE

Ice Cream Questions



- 1. What is the most popular flavor?
- 2. What is the least popular flavor?
- 3. Are you surprised by these?
- 4. What's your favorite flavor?



Graph Reading

- 1. What was the population in the year 1880?
- 2. When was the population 250 million?
- 3. When was the highest population?
- 4. What happened around 1930–1940?

Restate the Sentences

Rewrite the sentences keeping the general meaning the same.



Chapter 8: Measurement

Convert the Height



- 5. Saori Kimura is 185 cm tall. What's that in feet and inches?_____
- 6. Hello Kitty stands five apples tall. A standard-size apple in England is 3.8 inches tall. How tall is she in centimeters?



Temperature

In the U.S., people talk about temperature in degrees Fahrenheit (°F). Around the world, many people use degrees Celcius (°C). You can convert using this formula.

$$f = \frac{9}{5}c + 32$$

- 1. What is 0°C in °F?
- 2. What is 0°F in °C?
- 3. What is 100°C in °F?
- 4. When is the temperature the same in both °F and °C?



True/False Quiz

1.	Water freezes at 0°C.	True /	FALSE
2.	Water boils at 100°C.	True /	False
3.	A football field is around 6,000 square meters.	True /	False
4.	The Eiffel Tower is 524 meters high.	True /	False
5.	It is 845 miles (1,360 kilometers) from New York to Los Angeles.	True /	False
6.	The volume of an average American bathtub is around 70 liters.	True /	False
7.	There are around 525,600 minutes in a year.	True /	False

Percent correct:



Number Prefixes

Many words start like this...

1	2	3	4	5	6	7	8
uni	bi	tri	quad	quin	sexa	septem	octo
mono	di	tri	tetra	penta	hexa	hepta	octa



Brainstorming

Think of some words that begin with these prefixes.

<u>bi-</u>	<u>tri-</u>	<u>oct-</u>
	<u>bi-</u>	<u>bi-</u> <u>tri-</u>

Demonstrations II: Statistics

RULES

In pairs or individually, make and give a math demonstration. Presentations should be two to three minutes long. Explain the topic and show examples on the chalk board.

KEY POINTS

Clarity • Volume • Pronunciation • Chalkboard • Time

TOPICS

- A) Why is October the tenth month?
- B) What are your classmates' favorite colors?
- C) How many students are in each class at this school?
- D) How many teachers teach each subject at this school?
- E) How many of each type of class do you have each week?
- F) How long does it take your classmates to come to school?
- G) How many hours of sleep do your classmates get each night?
- H) How many rooms on there on each floor of the school building?
- I) How many staircases are there, and how many steps do they have?
- J) How much time do your classmates spend each day watching TV?



A projector. Photo by Dave Pape.

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